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# PRINCIPLES

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# OF CONVOLUTIONAL

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# DECODING

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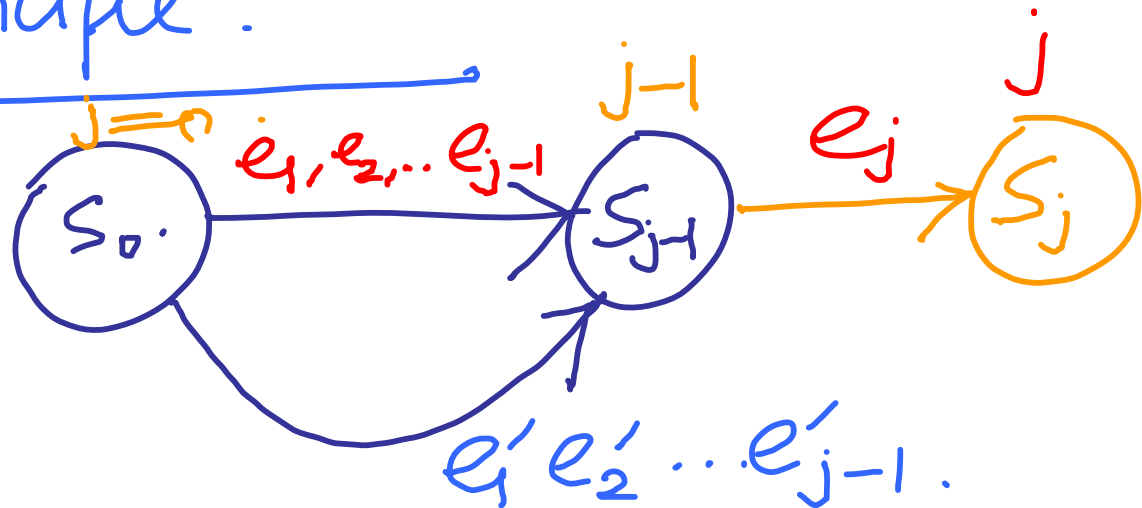
Key ideas to  
decode received  
sequence.

✓ Maximum Likelihood Decoder  
⇒ Minimum Distance

Decoding  
Finding  
Path Through Trellis  
State Sequence.

$d_H(\bar{r}, \bar{c})$   
Hamming Distance.  
Decoded codeword.  
received sequence.

1: Principle:



Let minimum distance from  
state  $s_0$  to state  $s_j$

$$\begin{array}{c} \text{min distance} \\ \text{path} \end{array} = \frac{e_1 e_2 \dots e_{j-1} e_j}{d_{\min}}$$

Then,  $e_1 e_2 \dots e_{j-1}$  is the  
minimum distance path from  
 $s_0$  to  $s_{j-1}$ .

Proof: Proof by contradiction  
If there exists an alternative  
path  $e'_1 e'_2 \dots e'_{j-1}$  which  
has min distance from  $s_0$  to  $s_{j-1}$

Then,

$$= d_{\min}(e_1, e_2, e_3 \dots, e_j)$$

$$= \sum_{i=1}^j d(e_i)$$

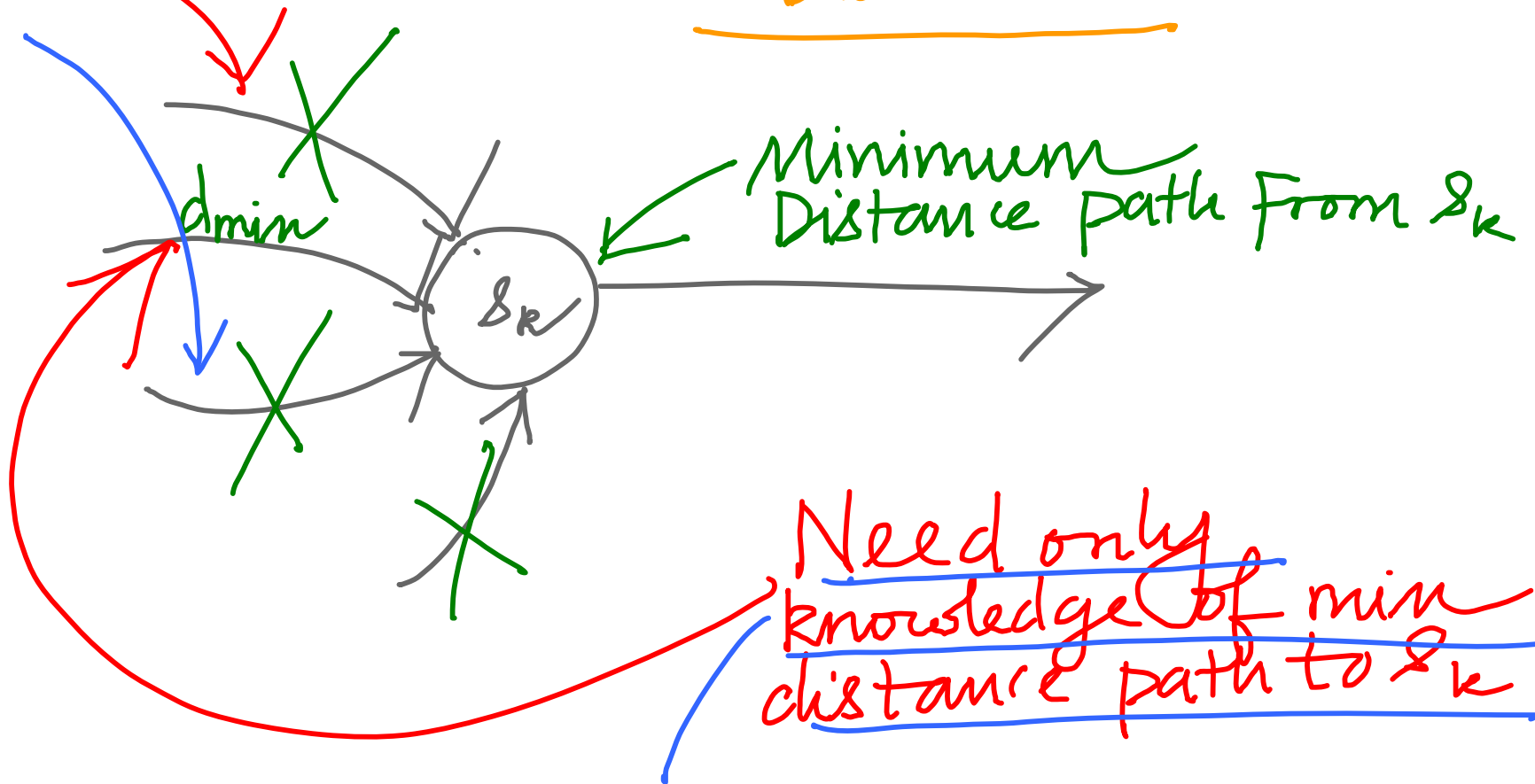
$$= \underbrace{\sum_{i=1}^{j-1} d(e_i) + d(e_j)}_0$$

$$\geq \sum_{i=1}^{j-1} d(e'_i) + d(e_j)$$

$$= d(e'_1, e'_2 \dots e'_{j-1}, e_j)$$

$d_{\min} \geq d(e'_1 e'_2 \dots e'_{j-1} e_j)$   
Contradiction!  
minimum Distance.

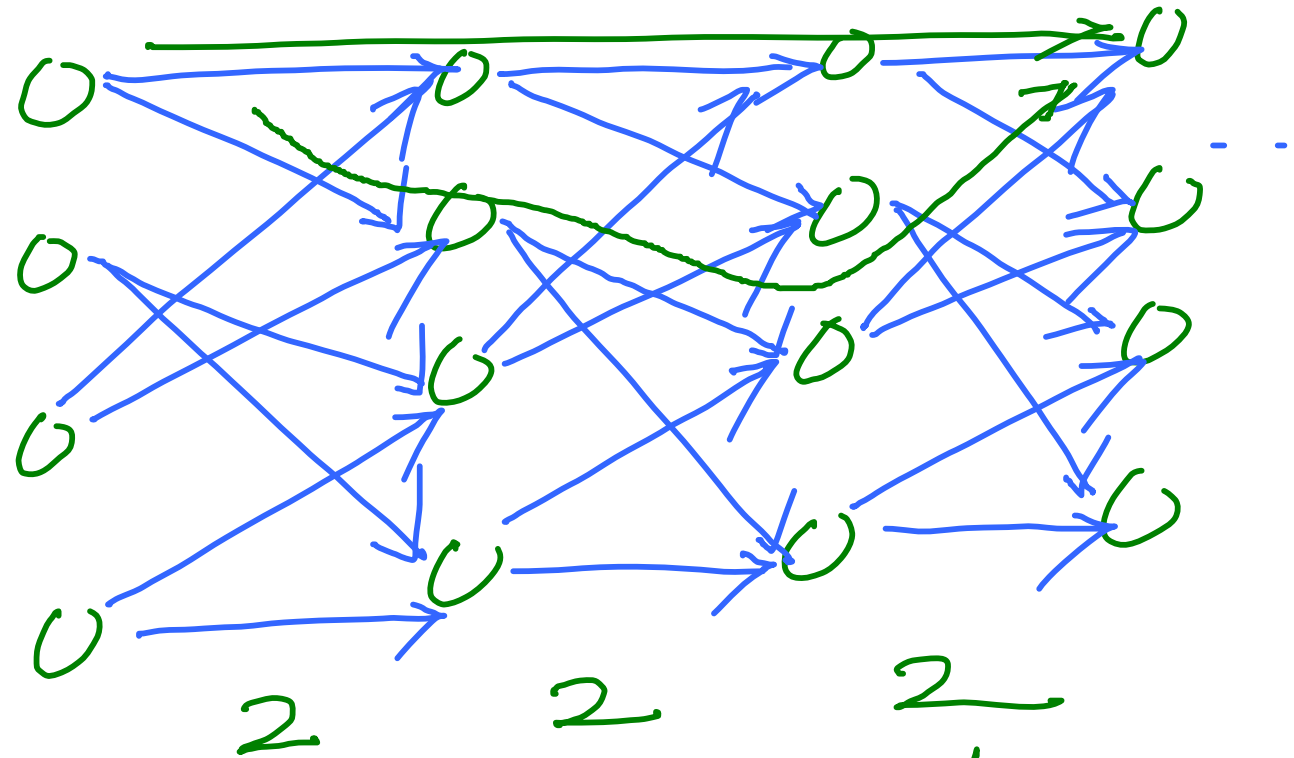
Ignore rest.



Need only knowledge of min distance path to  $s_k$

↓ No need to store any other information

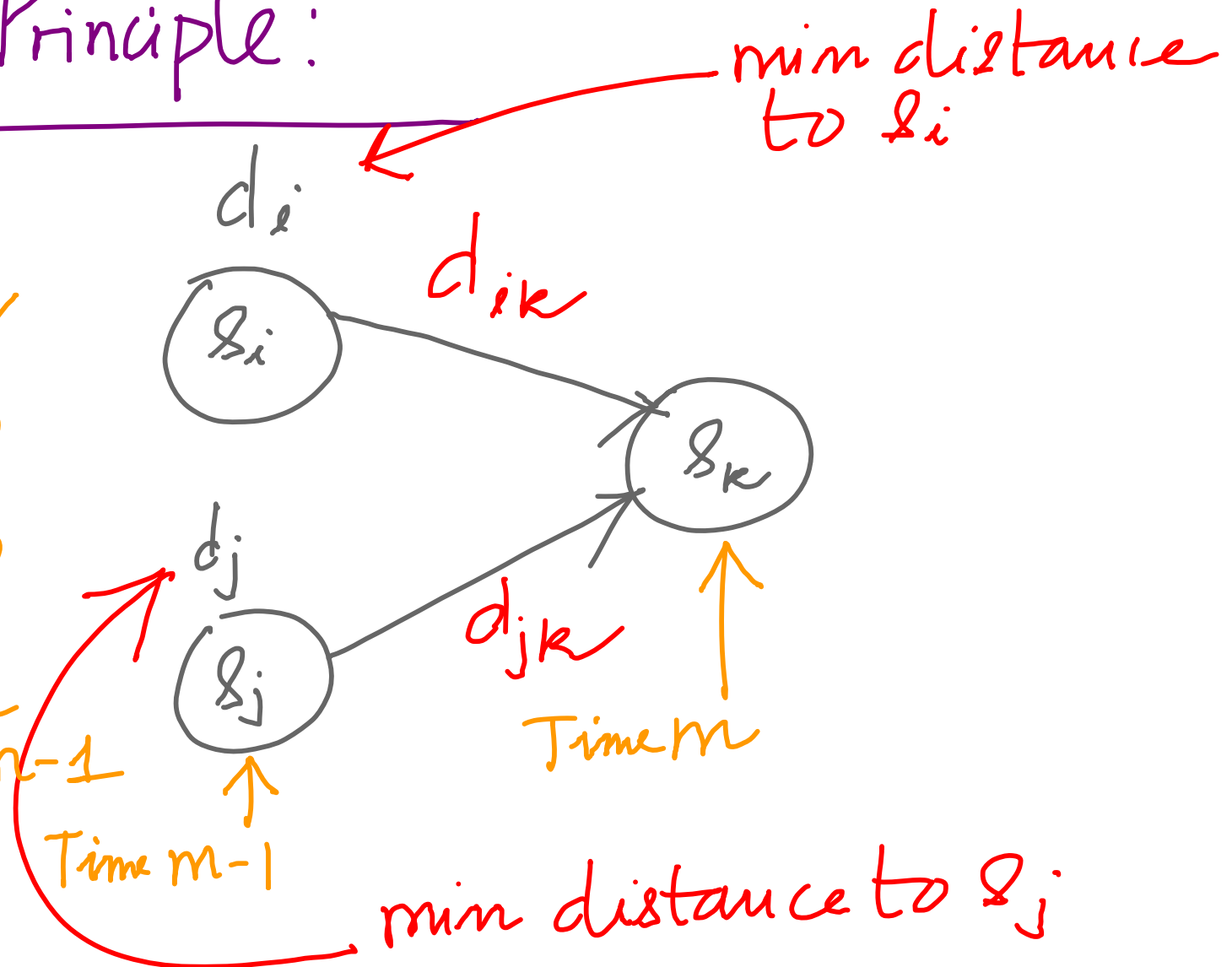
↓ This principle greatly simplifies computation.



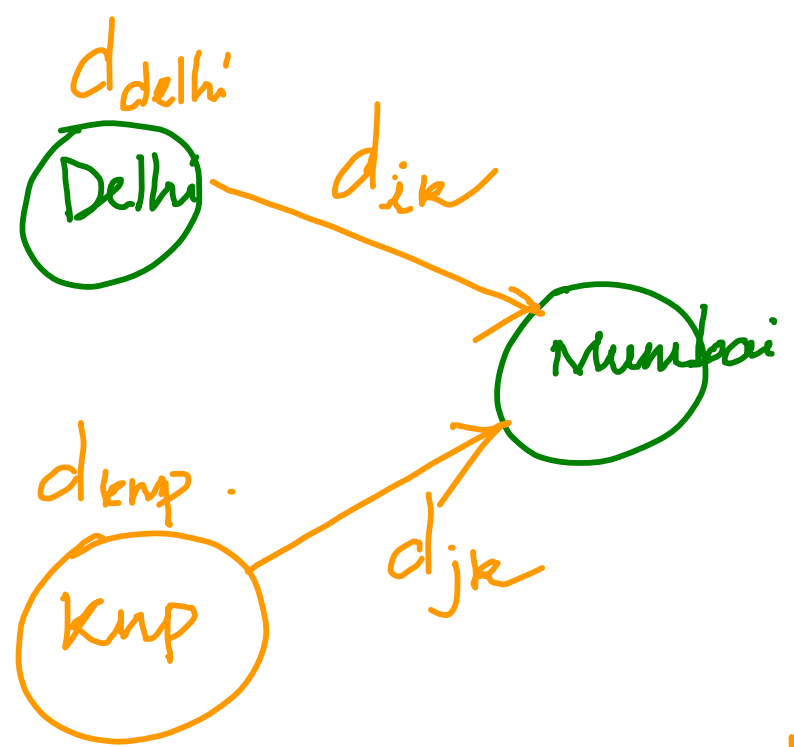
Total # paths becomes exponential!  
Very difficult (impossible) to compute

## # 2 Principle:

Finding min distance at time  $m$ , given minimum distance of nodes at time  $m-1$



$$\text{min distance to } s_k \\ = \min \{ d_i + d_{ik}, d_j + d_{jk} \}$$



$$C_{mum} = \min \left\{ \begin{array}{l} C_{delhi} + d_{ik} \\ d_{knp} + d_{jk} \end{array} \right\}$$